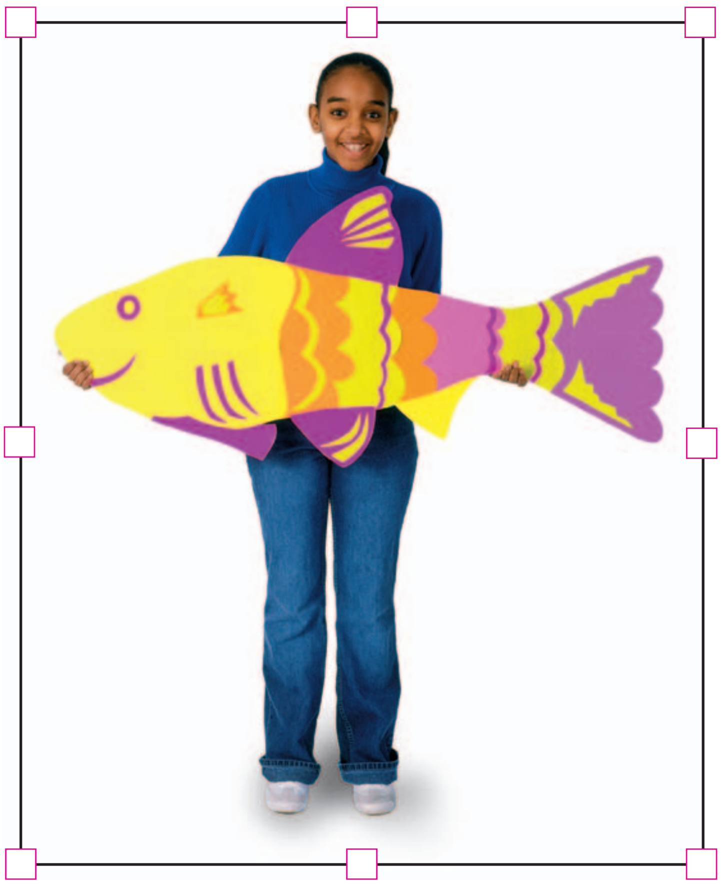
Investigation 4

Similarity and Ratios

You can enhance a report or story by adding photographs, drawings, or diagrams. Once you place a graphic in an electronic document, you can enlarge, reduce, or move it. In most programs, clicking on a graphic causes it to appear inside a frame with buttons along the sides, like the figure below.

You can change the size and shape of the image by grabbing and dragging the buttons.



Here are examples of the image after it has been resized.


Left


Middle


Right

Getting Ready for Problem 4.1

- How do you think this technique produced these variations of the original shape?
- Which of these images appears to be similar to the original? Why?

One way to describe and compare shapes is by using **ratios.** A ratio is a comparison of two quantities such as two lengths. The original figure is about 10 centimeters tall and 8 centimeters wide. You say, "the *ratio* of height to width is 10 to 8."

This table gives the ratio of height to width for the images.

Image Information

Figure	Height (cm)	Width (cm)	Height to Width Ratio
Original	10	8	10 to 8
Left	8	3	8 to 3
Middle	3	6	3 to 6
Right	5	4	5 to 4

- What do you observe about the ratios of height to width in the similar figures?

The comparisons "10 to 8" and "5 to 4" are **equivalent ratios.** Equivalent ratios name the same number. In both cases, if you write the ratio of height to width as a decimal, you get the same number.

$$10 \div 8 = 1.25 \qquad\qquad 5 \div 4 = 1.25$$

The same is true if you write the ratio of width to height as a decimal.

"8 to 10"   "4 to 5"

$$8 \div 10 = 0.8 \qquad\qquad 4 \div 5 = 0.8$$

Equivalence of ratios is a lot like equivalence of fractions. In fact, ratios are often written in the form of fractions. You can express equivalent ratios with equations like these:

$$\frac{10}{8} = \frac{5}{4}$$

$$\frac{8}{10} = \frac{4}{5}$$

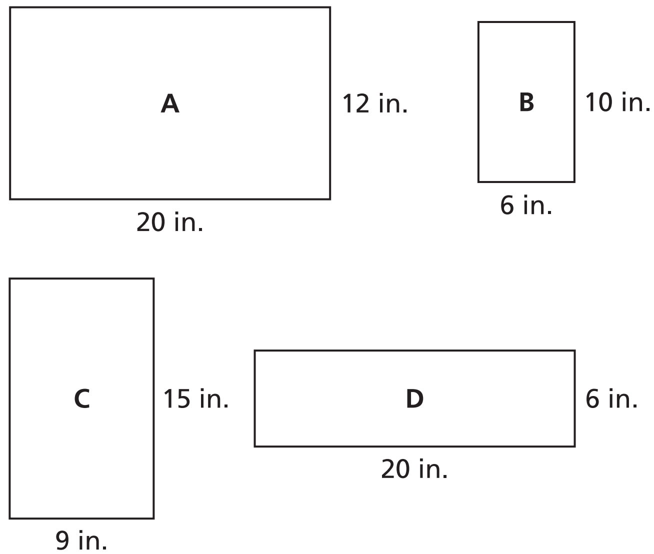
4.1 Ratios Within Similar Parallelograms

When two figures are similar, you know there is a scale factor that relates each length in one figure to the corresponding length in the other. You can also find a ratio between any two lengths in a figure. This ratio will describe the relationship between the corresponding lengths in a similar figure. You will explore this relationship in the next problem.

When you work with the diagrams in this investigation, assume that all measurements are in centimeters. Many of the drawings are not shown at actual size.

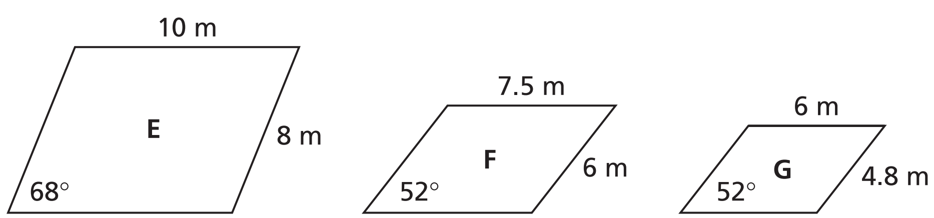
Problem 4.1 Ratios Within Similar Parallelograms

A. The lengths of two sides are given for each rectangle.



1. For each rectangle, find the ratio of the length of a short side to the length of a long side.
2. What do you notice about the ratios in part (1) for similar rectangles? About the ratios for non-similar rectangles?
3. For two similar rectangles, find the scale factor from the smaller rectangle to the larger rectangle. What information does the scale factor give about two similar figures?
4. Compare the information given by the scale factor to the information given by the ratios of side lengths.

B. **1.** For each parallelogram, find the ratio of the length of a longer side to the length of a shorter side. How do the ratios compare?



2. Which of the parallelograms are similar? Explain.

C. If the ratio of adjacent side lengths in one parallelogram is equal to the ratio of the corresponding side lengths in another, can you say that the parallelograms are similar? Explain.

ACE **Homework starts on page 66.**

4.2 Ratios Within Similar Triangles

Since all rectangles contain four 90° angles, you can show that rectangles are similar just by comparing side lengths. You now know two ways to show that rectangles are similar.

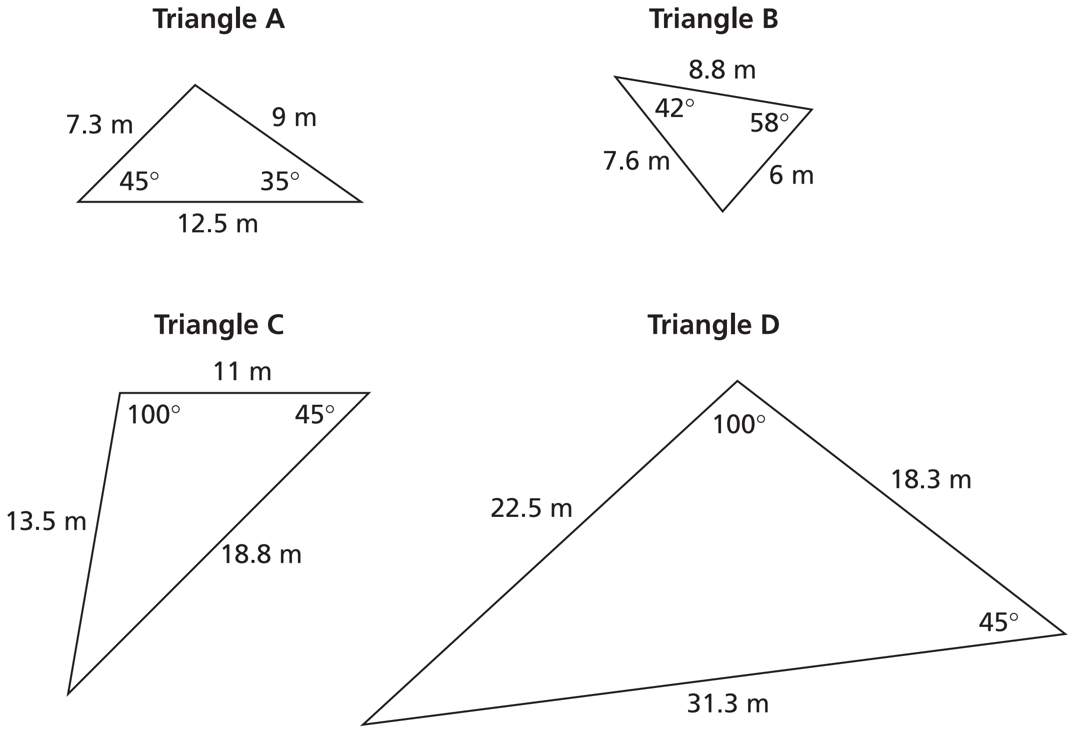
(1) Show that the scale factors between corresponding side lengths are equal. (compare length to length and width to width)

(2) Show that the ratios of corresponding sides within each shape are equal. (compare length to width in one rectangle and length to width in the other)

However, comparing only side lengths of a non-rectangular parallelogram or a triangle is not enough to understand its shape. In this problem, you will use angle measures and side-length ratios to find similar triangles.

Problem 4.2 Ratios Within Similar Triangles

For Questions A and B, use the triangles below. Side lengths are approximate.



A. Identify the triangles that are similar to each other. Explain how you use the angles and sides to identify the similar triangles.

B. **1.** Within each triangle, find the ratio of shortest side to longest side. Find the ratio of shortest side to "middle" side.

2. How do the ratios of side lengths compare for similar triangles?

3. How do the ratios of side lengths compare for triangles that are *not* similar?

ACE **Homework starts on page 66.**

4.3 Finding Missing Parts

When you know that two figures are similar, you can find missing lengths in two ways.

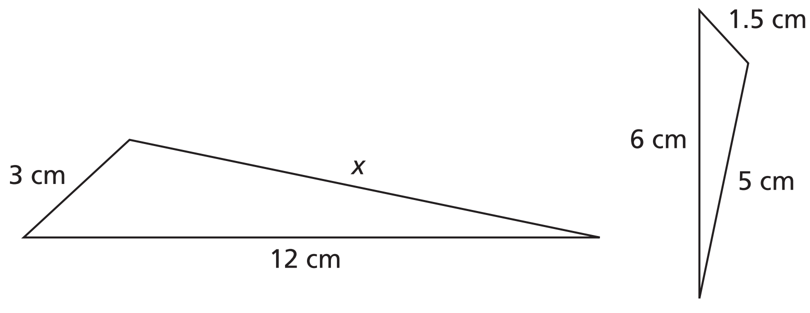
(1) Use the scale factor from one figure to the other.

(2) Use the ratios of the side lengths within each figure.

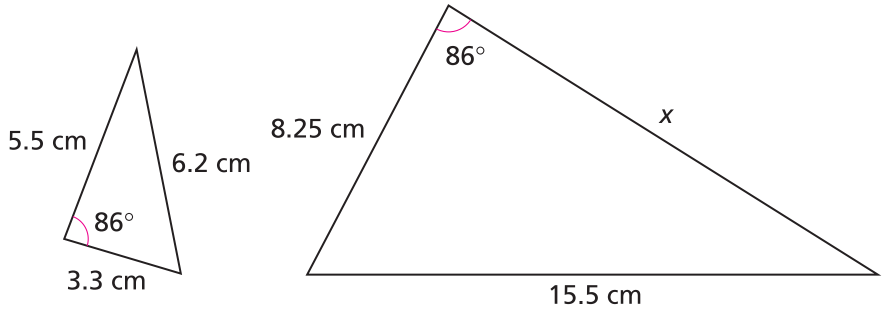
Problem 4.3 Using Similarity to Find Measurements

For Questions A–C, each pair of figures is similar. Find the missing side lengths. Explain.

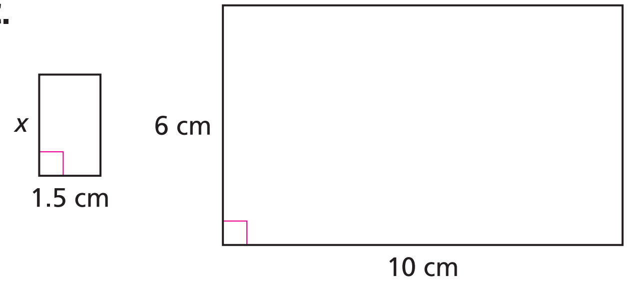
A.



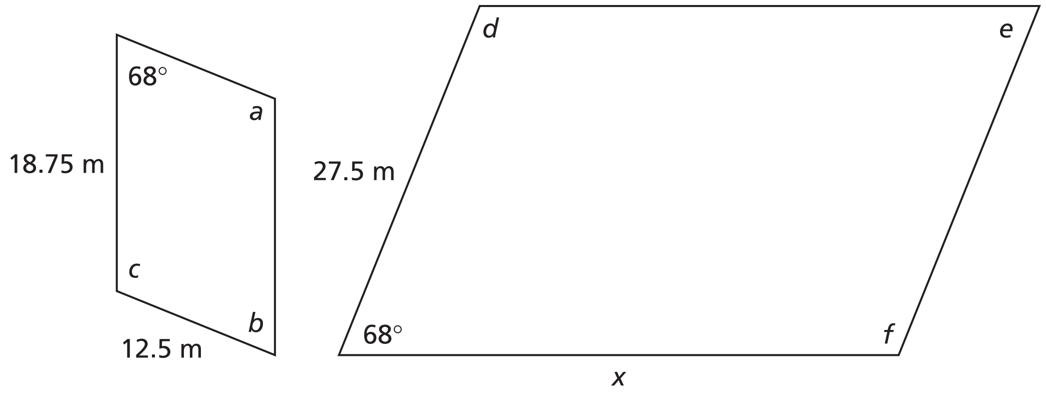
B.



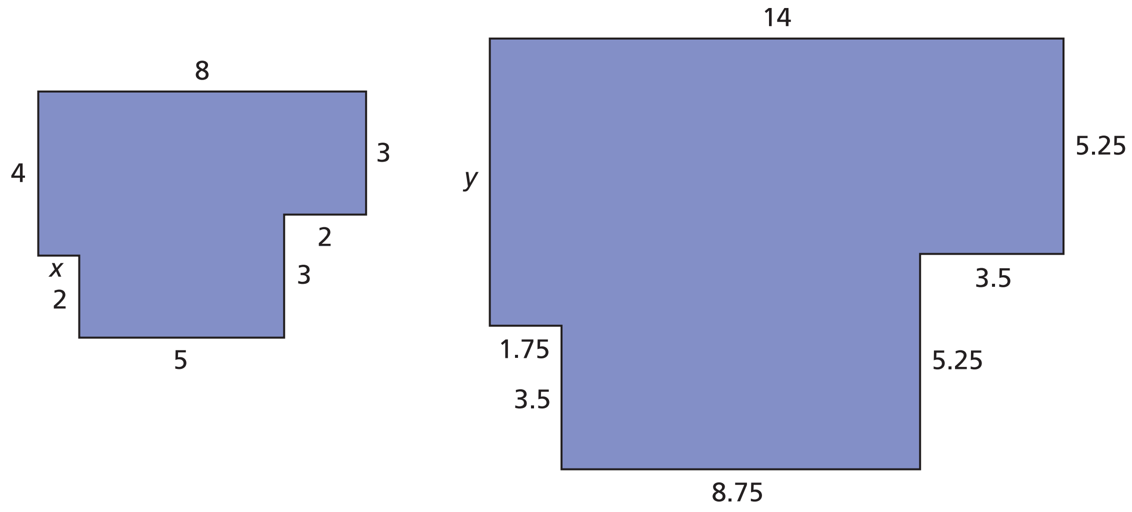
C.



D. The figures are similar. Find the missing measurements. Explain.



E. The figures below are similar. The measurements shown are in inches.



1. Find the value of x using ratios.
2. Find the value of y using scale factors.
3. Find the area of one of the figures.
4. Use your answer to part (3) and the scale factor. Find the area of the other figure. Explain.

ACE **Homework starts on page 66.**