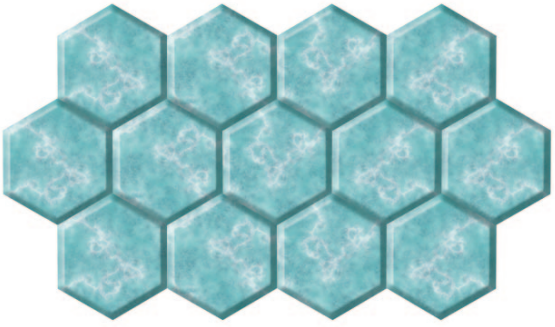
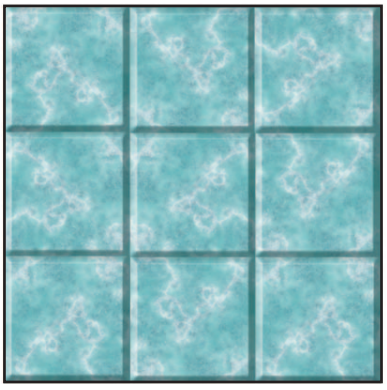
Investigation 3

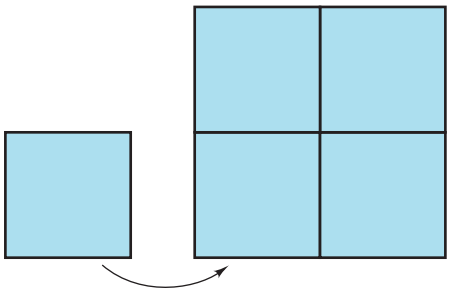
Similar Polygons

In *Shapes and Designs*, you learned that some polygons can fit together to cover, or tile, a flat surface. For example, the surface of a honeycomb is covered with a pattern of regular hexagons. Many bathroom and kitchen floors are covered with a pattern of square tiles.





If you look closely at the pattern of squares on the right above, you can see that the large square, which consists of nine small squares, is similar to each of the nine small squares. The nine-tile square has sides made of three small squares, so the scale factor from the small square to the nine-tile square is 3. You can also take four small squares and put them together to make a four-tile square that is similar to the nine-tile square. The scale factor in this case is 2.



Similar; scale factor is 2.

However, no matter how closely you look at the hexagon pattern, you cannot find a large hexagon made up of similar smaller hexagons.

If congruent copies of a shape can be put together to make a larger, similar shape, the original shape is called a **rep-tile.** A square is a rep-tile, but a regular hexagon is not.

3.1 Rep-Tile Quadrilaterals

In the next problem, you will see if rectangles and non-rectangular quadrilaterals are also rep-tiles.

Problem 3.1 Forming Rep-Tiles With Similar Quadrilaterals

Sketch and make several copies of each of the following shapes:

- a non-square rectangle
- a non-rectangular parallelogram
- a trapezoid

A. Which of these shapes can fit together to make a larger shape that is similar to the original? Make a sketch to show how the copies fit together.

B. Look at your sketches from Question A.

1. What is the scale factor from the original figure to the larger figure? Explain.

2. How does the perimeter of the new figure relate to the perimeter of the original?

3. How does the area of the new figure relate to the area of the original?

C. 1. Extend the rep-tile patterns you made in Question A. Do this by adding copies of the original figure to make larger figures that are similar to the original.

2. Make sketches showing how the figures fit together.

3. Find the scale factor from each original figure to each new figure. Explain.

4. Explain what the scale factor indicates about the corresponding side lengths, perimeters, and areas.

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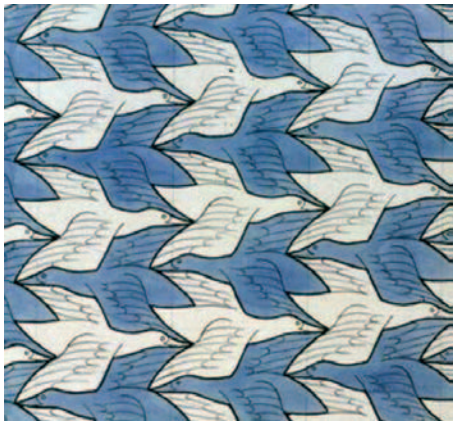
3.2 Rep-Tile Triangles

While rep-tiles must tessellate, not every shape that tessellates is a rep-tile.

Are the birds in the tessellation below rep-tiles?



All triangles tessellate. Are all triangles rep-tiles?

Problem 3.2 Forming Rep-Tiles With Similar Triangles

Sketch and make several copies of each of the following shapes:

- a right triangle
- an isosceles triangle
- a scalene triangle

A. Which of these triangles fit together to make a larger triangle that is similar to the original? Make a sketch to show how the copies fit together.

B. Look at your sketches from Question A.

1. What is the scale factor from each original triangle to each larger triangle? Explain.

2. How is the perimeter of the new triangle related to the perimeter of the original?

3. How is the area of the new triangle related to the area of the original?

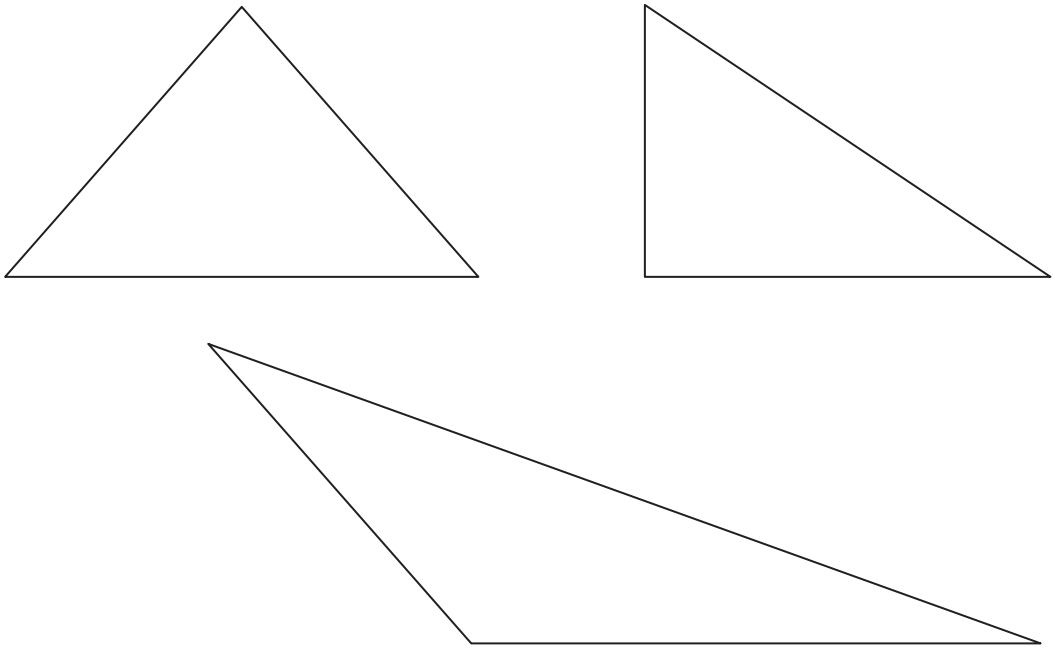
C. **1.** Extend the rep-tile patterns you made in Question A. Do this by adding copies of the original triangle to make larger triangles that are similar to the original.

2. Make sketches to show how the triangles fit together.

3. Find the scale factor from each original triangle to each new triangle. Explain.

4. Explain what the scale factor indicates about the corresponding side lengths, perimeters, and areas.

D. Study the rep-tile patterns. See if you can find a strategy for dividing each of the triangles below into four or more similar triangles. Make sketches to show your ideas.



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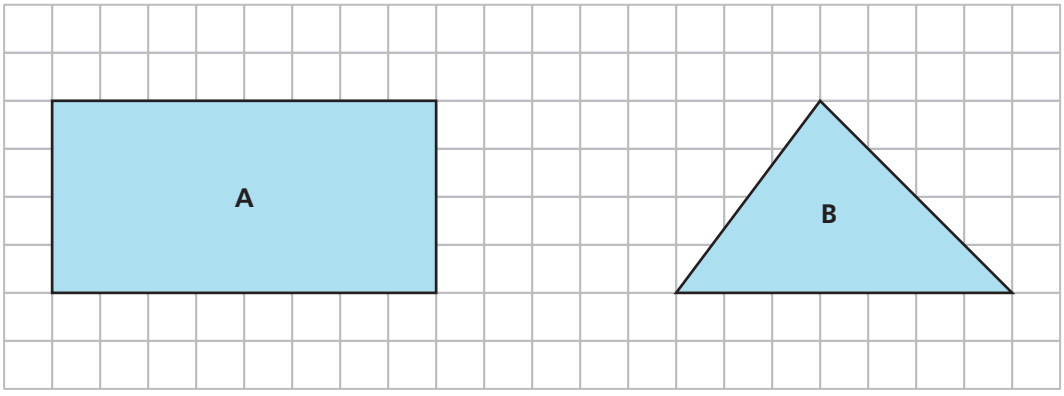
3.3 Scale Factors and Similar Shapes

You know that the scale factor from one figure to a similar figure gives you information about how the side lengths, perimeters, and areas of the figures are related. You will use what you learned in the next problem.

Problem 3.3 Scale Factors and Similar Shapes

For Questions A and B, use the two figures on the grid.



A. For parts (1)–(3), draw a rectangle similar to rectangle A that fits the given description. Find the base and height of each new rectangle.

1. The scale factor from rectangle A to the new rectangle is 2.5.

2. The area of the new rectangle is $\frac{1}{4}$ the area of rectangle A.

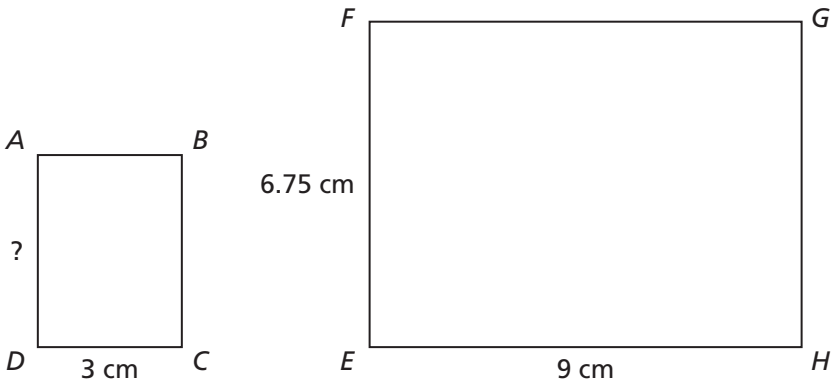
3. The perimeter of the new rectangle is three times the perimeter of rectangle A.

B. For parts (1)–(2), draw a triangle similar to triangle B that fits the given description. Find the base and height of each new triangle.

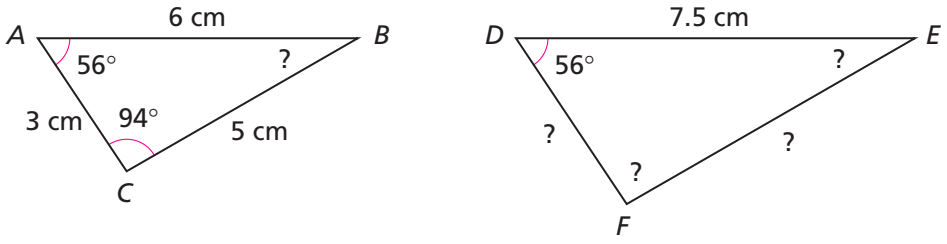
1. The area of the new triangle is nine times the area of triangle B.

2. The scale factor from triangle B to the new triangle is $\frac{1}{2}$.

C. 1. Rectangles $ABCD$ and $EFGH$ are similar. Find the length of side AD. Explain.



2. Triangles ABC and DEF are similar.



a. By what number do you multiply the length of side AB to get the length of side DE?

b. Find the missing side lengths and angle measures. Explain.

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