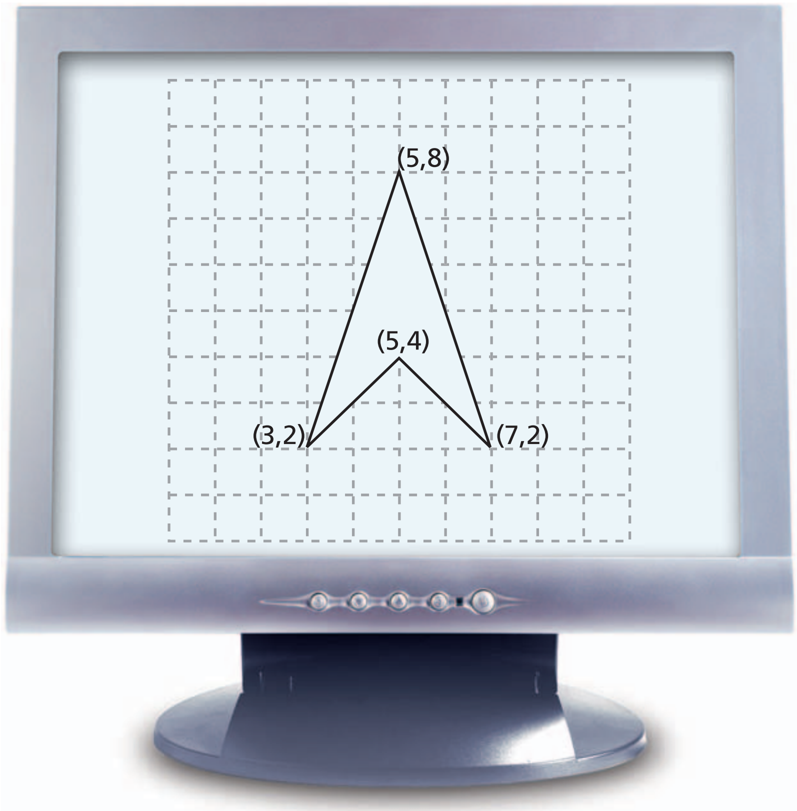
Investigation 2

Similar Figures

Zack and Marta want to design a computer game that involves several animated characters. Marta asks her uncle Carlos, a programmer for a video game company, about computer animation.

Carlos explains that the computer screen can be thought of as a grid made up of thousands of tiny points, called pixels. To animate a figure, you need to enter the coordinates of key points on the figure. The computer uses these key points to draw the figure in different positions.



Sometimes the figures in a computer game need to change size. A computer can make a figure larger or smaller if you give it a rule for finding key points on the new figure, using key points from the original figure.

2.1 Drawing Wumps

Zack and Marta's computer game involves a family called the Wumps. The members of the Wump family are various sizes, but they all have the same shape. That is, they are similar. Mug Wump is the game's main character. By enlarging or reducing Mug, a player can transform him into other Wump family members.

Zack and Marta experiment with enlarging and reducing figures on a coordinate grid. First, Zack draws Mug Wump on graph paper. Then, he labels the key points from *A* to *X* and lists the coordinates for each point. Marta writes the rules that will transform Mug into different sizes.

Problem 2.1 Making Similar Figures

Marta tries several rules for transforming Mug into different sizes. At first glance, all the new characters look like Mug. However, some of the characters are quite different from Mug.

A. To draw Mug on a coordinate graph, refer to the "Mug Wump" column in the table on the next page. For parts (1)–(3) of the figure, plot the points in order. Connect them as you go along. For part (4), plot the two points, but do not connect them. When you are finished, describe Mug's shape.

B. In the table, look at the columns for Zug, Lug, Bug, and Glug.

1. For each character, use the given rule to find the coordinates of the points. For example, the rule for Zug is $(2x, 2y)$. This means that you multiply each of Mug's coordinates by 2. Point *A* on Mug is (0, 1), so the corresponding point on Zug is (0, 2). Point *B* on Mug is (2, 1), so the corresponding point *B* on Zug is (4, 2).
2. Draw Zug, Lug, Bug, and Glug on separate coordinate graphs. Plot and connect the points for each figure, just as you did to draw Mug.

C. 1. Compare the characters to Mug. Which are the impostors?
2. What things are the same about Mug and the others?
3. What things are different about the five characters?

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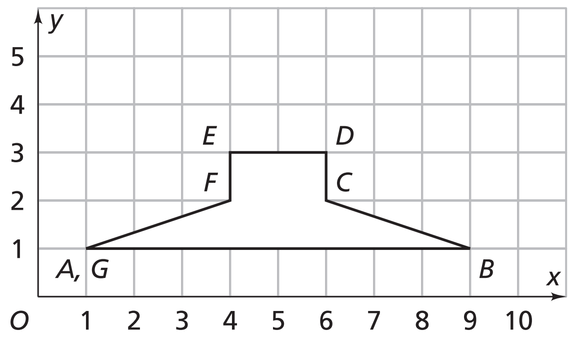
Coordinates of Game Characters

	Mug Wump	Zug	Lug	Bug	Glug
Rule	**(*x*, *y*)**	**(2*x*, 2*y*)**	**(3*x*, *y*)**	**(3*x*, 3*y*)**	**(*x*, 3*y*)**
Point	**Part 1**				
A	(0, 1)	(0, 2)			
B	(2, 1)	(4, 2)			
C	(2, 0)				
D	(3, 0)				
E	(3, 1)				
F	(5, 1)				
G	(5, 0)				
H	(6, 0)				
I	(6, 1)				
J	(8, 1)				
K	(6, 7)				
L	(2, 7)				
M	(0, 1)				
	Part 2 (Start Over)				
N	(2, 2)				
O	(6, 2)				
P	(6, 3)				
Q	(2, 3)				
R	(2, 2)				
	Part 3 (Start Over)				
S	(3, 4)				
T	(4, 5)				
U	(5, 4)				
V	(3, 4)				
	Part 4 (Start Over)				
W	(2, 5) (make a dot)				
X	(6, 5) (make a dot)				

2.2 Hats Off to the Wumps

Zack experiments with multiplying Mug's coordinates by different whole numbers to make other characters. Marta asks her uncle how multiplying the coordinates by a decimal or adding numbers to or subtracting numbers from each coordinate will affect Mug's shape. He gives her a sketch for a new shape (a hat for Mug) and some rules to try.

Mug's Hat



Problem 2.2 Changing a Figure's Size and Location

A. Look at the rules for Hats 1–5 in the table. Before you find any coordinates, predict how each rule will change Mug's hat.

B. Copy and complete the table. Give the coordinates of Mug's hat and the five other hats. Plot each new hat on a separate coordinate grid and connect each point as you go.

Rules for Mug's Hat

	Mug's Hat	Hat 1	Hat 2	Hat 3	Hat 4	Hat 5
Point	(x, y)	$(x + 2, y + 3)$	$(x - 1, y + 4)$	$(x + 2, 3y)$	$(0.5x, 0.5y)$	$(2x, 3y)$
A	(1, 1)					
B	(9, 1)					
C						
D						
E						
F						
G						

C. **1.** Compare the angles and side lengths of the hats.

2. Which hats are similar to Mug's hat? Explain why.

D. Write rules that will make hats similar to Mug's in each of the following ways.

1. The side lengths are one third as long as Mug's.

2. The side lengths are 1.5 times as long as Mug's.

3. The hat is the same size as Mug's, but has moved right 1 unit and up 5 units.

E. Write a rule that makes a hat that is *not* similar to Mug's.

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2.3 Mouthing Off and Nosing Around

How did you decide which of the computer game characters were members of the Wump family and which were imposters?

In general, how can you decide whether or not two shapes are similar?

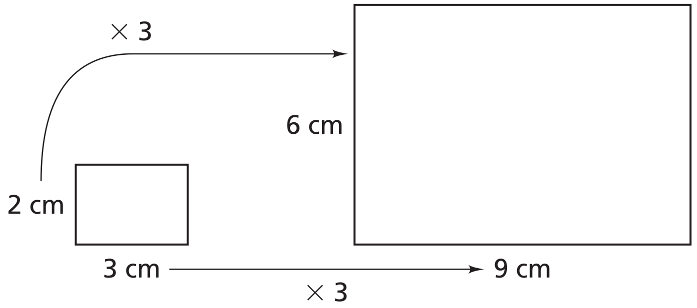
Your experiments with rubber-band stretchers, copiers, and coordinate plots suggest that for two figures to be **similar,** there must be the following correspondence between the figures.

- The side lengths of one figure are multiplied by the same number to get the corresponding side lengths in the second figure.
- The corresponding angles are the same size.

The number that the side lengths of one figure can be multiplied by to give the corresponding side lengths of the other figure is called the **scale factor.**

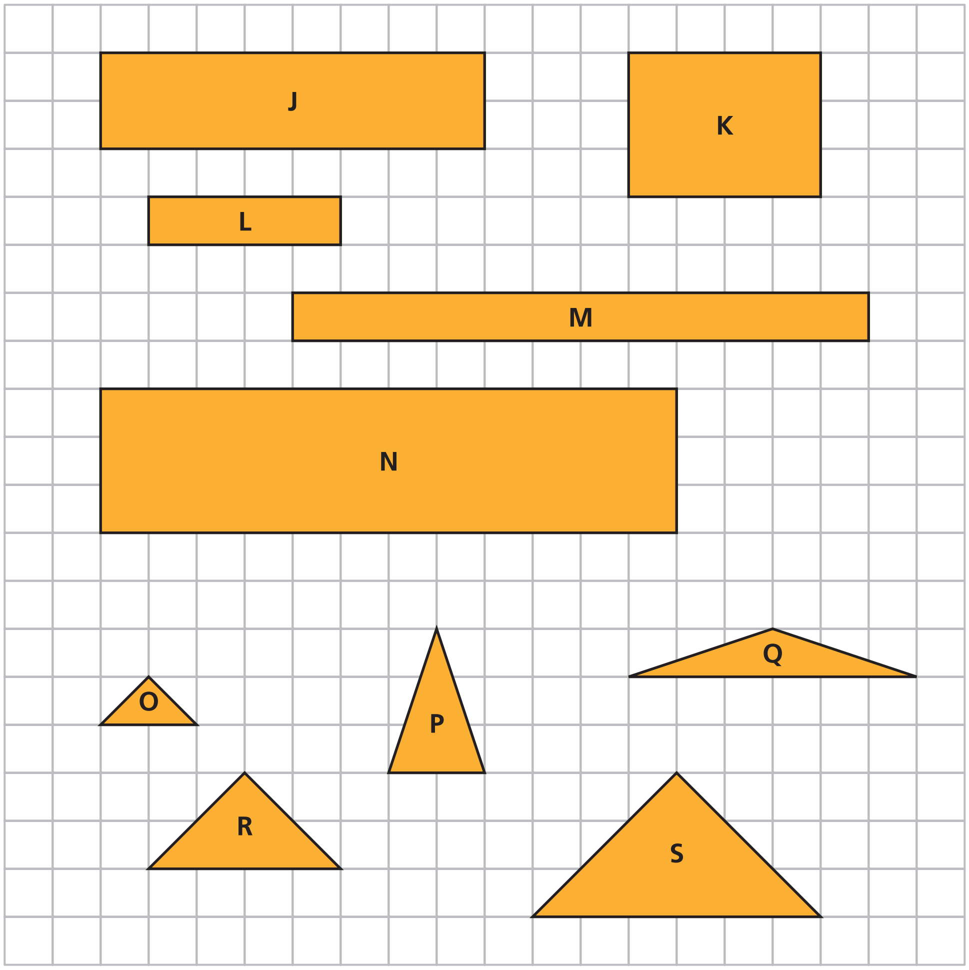
Getting Ready for Problem 2.3

The rectangles below are similar. The scale factor from the smaller rectangle to the larger rectangle is 3.



- What is the scale factor from the larger rectangle to the smaller rectangle?

The diagram shows a collection of mouths (rectangles) and noses (triangles) from the Wump family and from some impostors.



Problem 2.3 Scale Factors

A. After studying the noses and mouths in the diagram, Marta and Zack agree that rectangles J and L are similar. However, Marta says the scale factor is 2, while Zack says it is 0.5. Is either of them correct? How would you describe the scale factor so there is no confusion?

B. Decide which pairs of rectangles are similar and find the scale factor.

C. Decide which pairs of triangles are similar and find the scale factor.

D. **1.** Can you use the scale factors you found in Question B to predict the relationship between the perimeters for each pair of similar rectangles? Explain.

2. Can you use the scale factors in Question B to predict the relationship between the areas for each pair of similar rectangles? Explain.

E. For parts (1)–(3), draw the figures on graph paper.

1. Draw a rectangle that is similar to rectangle J, but is larger than any rectangle shown in the diagram. What is the scale factor from rectangle J to your rectangle?

2. Draw a triangle that is *not* similar to any triangle shown in the diagram.

3. Draw a rectangle that is *not* similar to any rectangle shown in the diagram.

F. Explain how to find the scale factor from a figure to a similar figure.

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Did You Know?

You can make figures and then rotate, slide, flip, stretch, and copy them using a computer graphics program. There are two basic kinds of graphics programs. Paint programs make images out of pixels (which is a short way of saying "picture elements"). Draw programs make images out of lines that are drawn from mathematical equations.

The images you make in a graphics program are displayed on the computer screen. A beam of electrons activates a chemical in the screen, called phosphor, to make the images appear on your screen. If you have a laptop computer with a liquid crystal screen, an electric current makes the images appear on the screen.