

# Investigation

# 4

## Exploring Slope

All of the patterns of change you have explored in this unit involved constant rates. For example, you worked with walking rates expressed as meters per second and pledge rates expressed as dollars per mile. In these situations, you found that the rate affects the following things:

- the steepness of the graph
- the coefficient,  $m$ , of  $x$  in the equation  $y = mx + b$
- how the  $y$ -values in the table change for each unit change in the  $x$ -values

In this investigation, you will explore another way to express the constant rate.

### 4.1

## Climbing Stairs

Climbing stairs is good exercise, so some athletes run up and down stairs as part of their training. The steepness of stairs determines how difficult they are to climb. By investigating the steepness of stairs you can find another important way to describe the steepness of a line.

### Getting Ready for Problem 4.1

Consider these questions about the stairs you use at home, in your school, and in other buildings.

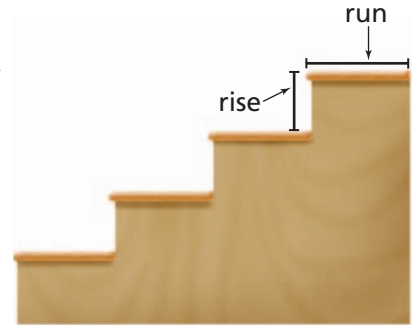
- How can you describe the steepness of the stairs?
- Is the steepness the same between any two consecutive steps?



Carpenters have developed the guidelines below to ensure that the stairs they build are relatively easy for a person to climb. Steps are measured in inches.

- The ratio of rise to run for each step should be between 0.45 and 0.60.
- The rise plus the run for each step should be between 17 and  $17\frac{1}{2}$  inches.

The steepness of stairs is determined by the ratio of the rise to the run for each step. The rise and run are labeled in the diagram at the right.



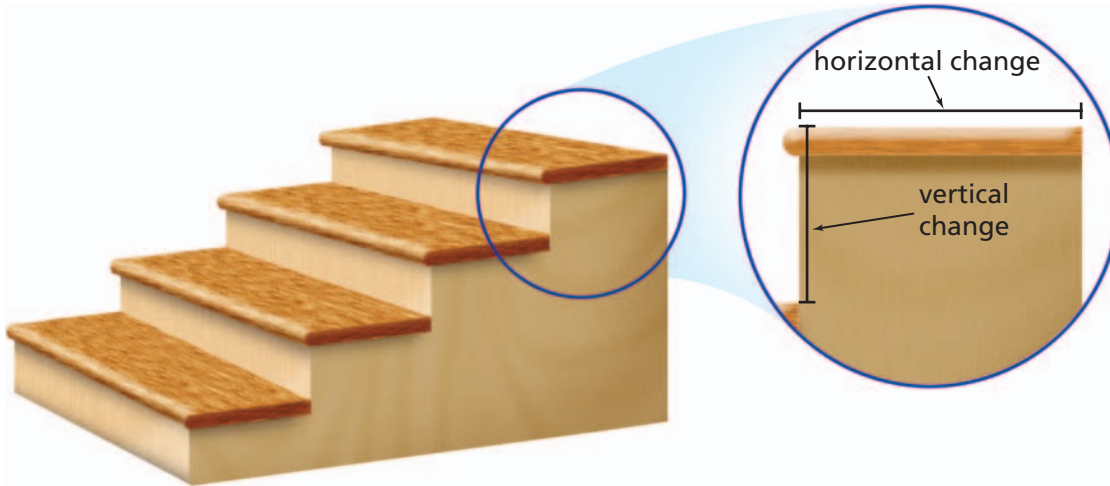
### Problem 4.1 Using Rise and Run

- A. 1.** Determine the steepness of a set of stairs in your school or home. To calculate the steepness you will need to
- measure the rise and run of at least two steps in the set of stairs.
  - make a sketch of the stairs, and label the sketch with the measurements you found.
  - find the ratio of rise to run.
- 2.** How do the stairs you measured compare to the carpenters' guidelines above?
- B.** A set of stairs is being built for the front of the new Arch Middle School. The ratio of rise to run is 3 to 5.
- 1.** Is this ratio within the carpenters' guidelines?
  - 2.** Make a sketch of a set of stairs that meet this ratio. Label the lengths of the rise and run of a step.
  - 3.** Sketch the graph of a line that passes through the origin and whose  $y$ -values change by 3 units for each 5-unit change in the  $x$ -values.
  - 4.** Write an equation for the line in part (3).
    - a.** What is the coefficient of  $x$  in the equation?
    - b.** How is the coefficient related to the steepness of the line represented by the equation?
    - c.** How is the coefficient related to the steepness of a set of stairs with this ratio?

**ACE** Homework starts on page 78.

## 4.2 Finding the Slope of a Line

The method for finding the steepness of stairs suggests a way to find the steepness of a line. A line drawn from the bottom step of a set of stairs to the top step touches each step in one point. The rise and the run of a step are the vertical and the horizontal changes, respectively, between two points on the line.



The steepness of the line is the ratio of rise to run, or vertical change to horizontal change, for this step. We call this ratio the **slope** of the line.

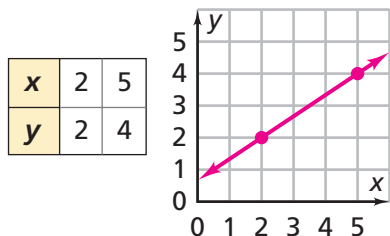
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} \text{ or } \frac{\text{rise}}{\text{run}}$$

Unlike the steepness of stairs, the slope of a line can be negative. To determine the slope of a line, you need to consider the direction, or sign, of the vertical and horizontal changes from one point to another. If vertical change is negative for positive horizontal change, the slope will be negative. Lines that slant *upward* from left to right have *positive slope*; lines that slant *downward* from left to right have *negative slope*.

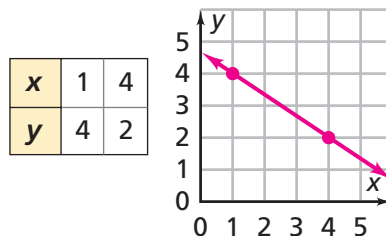
## Getting Ready for Problem 4.2

- For each graph, describe how you can find the slope of the line.

**Line With Positive Slope**



**Line With Negative Slope**



- The data in the table represent a linear relationship. Describe how you can find the slope of the line that represents the data.

<b>x</b>	-1	0	1	2	3	4
<b>y</b>	0	3	6	9	12	15

Information about a linear situation can be given in several different representations, such as a table, graph, equation, or verbal situation. These representations are useful in answering questions about linear situations.

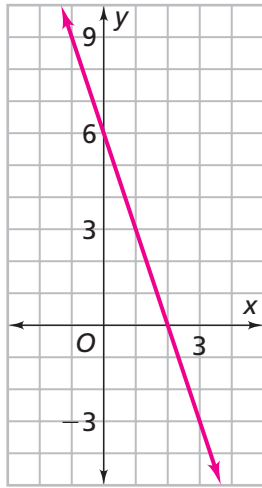
*How can we calculate the slope of a line from these representations?*



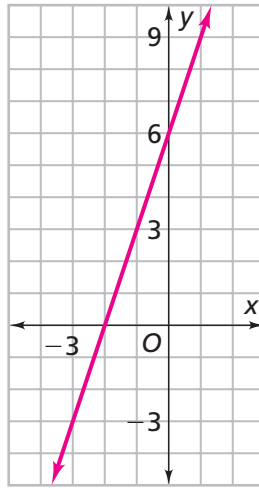
## Problem 4.2 Finding the Slope of a Line

A. The graphs, tables and equations all represent linear situations.

**Graph 1**



**Graph 2**



**Table 1**

<b>x</b>	-6	-4	-2	0	2	4
<b>y</b>	-10	-7	-4	-1	2	5

**Table 2**

<b>x</b>	1	2	3	4	5	6
<b>y</b>	4.5	4.0	3.5	3.0	2.5	2.0

**Equation 1**  
 $y = 2.5x + 5$

**Equation 2**  
 $y = 20 - 3x$

- Find the slope and y-intercept of the line represented in each situation.
  - Write an equation for each graph and table.
- B.** The points  $(3, 5)$  and  $(-2, 10)$  lie on a line. Find two more points that lie on this line. Explain your method.
- C.** Compare your methods for finding the slope of a line from a graph, table, and equation.

**ACE** Homework starts on page 78.

## 4.3

## Exploring Patterns With Lines

Your understanding of linear relationships can be used to explore some ideas about groups of lines.

## Getting Ready for Problem 4.3

The slope of a line is 3.

- Sketch a line with this slope.
- Can you sketch a different line with this slope? Explain.

## Problem 4.3 Exploring Patterns With Lines

**A.** Consider the two groups of lines shown below.

$$\text{Group 1: } y = 3x \quad y = 5 + 3x \quad y = 10 + 3x \quad y = -5 + 3x$$

$$\text{Group 2: } y = -2x \quad y = 4 - 2x \quad y = 8 - 2x \quad y = -4 - 2x$$

For each group:

1. What features do the equations have in common?
2. Graph each equation on the same coordinate axes. What patterns do you observe in the graphs?
3. Describe another group of lines that have the same pattern.

**B.** Consider the three pairs of lines shown below.

$$\begin{array}{lll} \text{Pair 1: } y = 2x & \text{Pair 2: } y = 4x & \text{Pair 3: } y = -3x + 5 \\ & y = -\frac{1}{2}x & y = -0.25x & y = \frac{1}{3}x - 1 \end{array}$$

For each pair:

1. What features do the equations have in common?
  2. Graph each equation on the same coordinate axes. What patterns do you observe in the graphs?
  3. Describe another pair of lines that have the same pattern.
- C.** Write equations for four lines that intersect to form the sides of a parallelogram. Explain what must be true about such lines.
- D.** Write equations for three lines that intersect to form a right triangle. Explain what must be true about such lines.
- E.** Describe how you can decide if two lines are parallel or perpendicular from the equations of the lines.

**ACE** Homework starts on page 78.

## 4.4 Pulling it All Together

Throughout this unit, you have learned several ways to represent linear relationships. You have also learned ways to move back and forth between these representations, tables, graphs, and equations to solve problems. The next problem pulls some of these ideas together.

### Problem 4.4 Writing Equations With Two Variables

#### A. Anjelita's Birthday

Today is Anjelita's birthday. Her grandfather gave Anjelita some money as a birthday gift. Anjelita plans to put her birthday money in a safe place and add part of her allowance to it each week. Her sister, Maria, wants to know how much their grandfather gave her and how much of her allowance she is planning to save each week. As usual, Anjelita does not answer her sister directly. Instead, she wants her to figure out the answer for herself. She gives her these clues:

- After five weeks, I will have saved a total of \$175.
  - After eight weeks, I will have saved \$190.
1. How much of her allowance is Anjelita planning to save each week?
  2. How much birthday money did Anjelita's grandfather give her for her birthday?
  3. Write an equation for the total amount of money  $A$  Anjelita will have saved after  $n$  weeks. What information do the  $y$ -intercept and coefficient of  $n$  represent in this context?

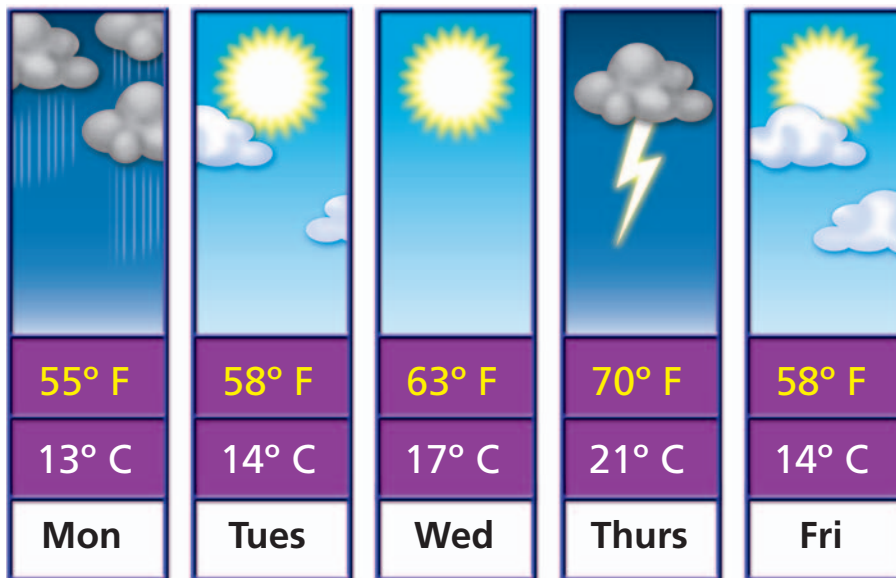


## B. Converting Temperatures

Detroit, Michigan, is just across the Detroit River from the Canadian city of Windsor, Ontario. Because Canada uses the Celsius temperature scale, weather reports in Detroit often give temperatures in both degrees Fahrenheit and in degrees Celsius. The relationship between degrees Fahrenheit and degrees Celsius is linear.

Two important reference points for temperature are:

- Water freezes at  $0^{\circ}\text{C}$ , or  $32^{\circ}\text{F}$ .
  - Water boils at  $100^{\circ}\text{C}$ , or  $212^{\circ}\text{F}$ .
1. Use this information to write an equation for the relationship between degrees Fahrenheit and degrees Celsius.
  2. How did you find the  $y$ -intercept? What does the  $y$ -intercept tell you about this situation?



**ACE** Homework starts on page 78.