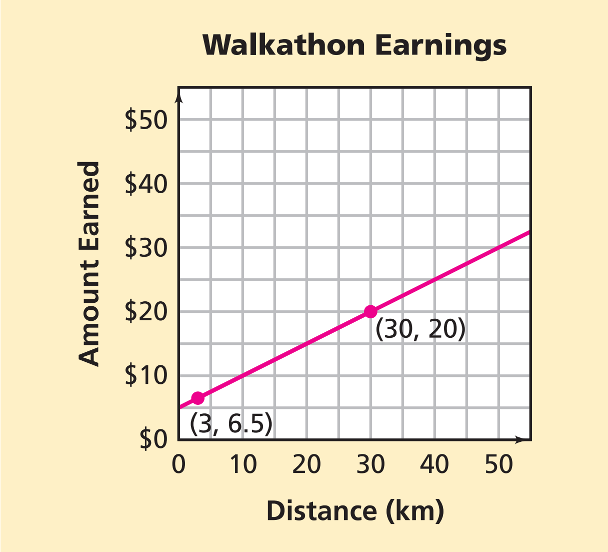
Investigation 3

Solving Equations

In the last investigation, you examined the patterns in the table and graph for the relationship between Alana's distance d and money earned A in the walkathon.

The equation $A = 5 + 0.5d$ is another way to represent the relationship between the distance and the money earned. The graph of this equation is a line that contains infinitely many points. The coordinates of the points on the line can be substituted into the equation to make a true statement.



For example, the point (3, 6.5) lies on the line. This means that $x = 3$ and $y = 6.5$. So, $6.5 = 5 + 0.5(3)$ is a true statement.

Similarly, the point (30, 20) lies on the line which means that $x = 30$ and $y = 20$, and $20 = 5 + 0.5(30)$ is a true statement.

We say that (3, 6.5) and (30, 20) are *solutions* to the equation $A = 5 + 0.5d$ because when the values for d and A are substituted into the equation we get a true statement. There are infinitely many solutions to $A = 5 + 0.5d$.

Because the corresponding entries in a table are the coordinates of points on the line representing the equation, we can also find a solution to an equation by using a table.

d	*A*
0	5
1	5.5
2	6
3	**6.5**
4	7
20	15
25	17.5
30	**20**

3.1 Solving Equations Using Tables and Graphs

In an equation with two variables, if the value of one variable is known, you can use a table or graph to find the value of the other variable. For example, suppose Alana raises $10 from a sponsor. Then you can ask: How many kilometers does Alana walk?

In the equation $A = 5 + 0.5d$, this means that $A = 10$. The equation is now $10 = 5 + 0.5d$.

Which value of d *will make this a true statement?*

Finding the value of *d* that will make this a true statement is called *solving the equation* for *d*.

Problem 3.1 Solving Equations Using Tables and Graphs

A. Use the equation $A = 5 + 0.5d$.

1. Suppose Alana walks 23 kilometers. Show how you can use a table and a graph to find the amount of money Alana gets from each sponsor.

2. Suppose Alana receives $60 from a sponsor. Show how you can use a table and a graph to find the number of kilometers she walks.

B. For each equation:

- Tell what information Alana is looking for.
- Describe how you can find the information.

1. $A = 5 + 0.5(15)$

2. $50 = 5 + 0.5d$

C. The following equations are related to situations that you have explored. Find the solution (the value of the variable) for each equation. Then, describe another way you can find the solution.

1. $D = 25 + 2.5(7)$

2. $70 = 25 + 2.5t$

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3.2 Exploring Equality

An equation states that two quantities are equal. In the equation $A = 5 + 0.5d$, A and $5 + 0.5d$ are the two quantities. Both represent the amount of money that Alana collects from each sponsor. Since each quantity represents numbers, you can use the properties of numbers to solve equations with one unknown variable.

Before we begin to solve linear equations, we need to look more closely at equality.

What does it mean for two quantities to be equal?

Let's look first at numerical statements.

Getting Ready for Problem 3.2

The equation 85 = 70 + 15 states that the quantities 85 and 70 + 15 are equal.

What do you have to do to maintain equality if you

- subtract 15 from the left-hand side of the equation?
- add 10 to the right-hand side of the original equation?
- divide the left-hand side of the original equation by 5?
- multiply the right-hand side of the original equation by 4?

Try your methods on another example of equality. Summarize what you know about maintaining equality between two quantities.

In the Kingdom of Montarek, the ambassadors carry diplomatic pouches. The contents of the pouches are unknown except by the ambassadors. Ambassador Milton wants to send one-dollar gold coins to another country.

$1 gold coin

diplomatic pouch

His daughter, Sarah, is a mathematician. She helps him devise a plan based on *equality* to keep track of the number of one-dollar gold coins in each pouch.

In each situation:

- Each pouch contains the same number of one-dollar gold coins.
- The number of gold coins on both sides of the equality sign is the same, but some coins are hidden in the pouches.

Try to find the number of gold coins in each pouch.

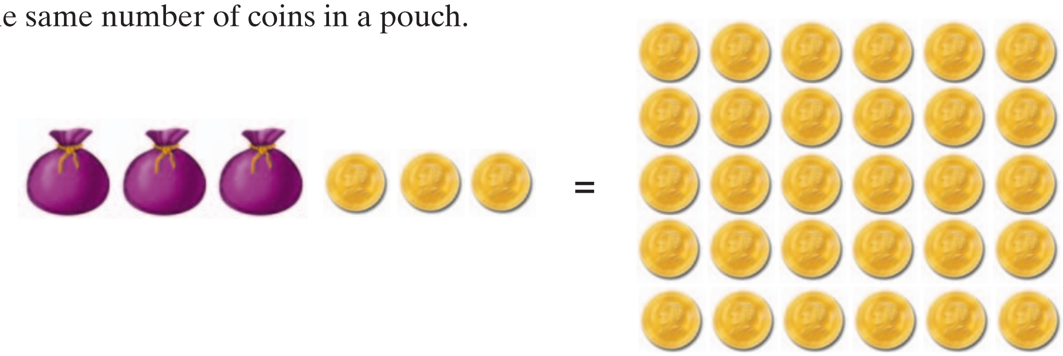
Problem 3.2 Exploring Equality

A. Sarah draws the following picture. Each pouch contains the same number of $1 gold coins.

How many gold coins are in each pouch? Explain your reasoning.

B. For each situation, find the number of gold coins in the pouch. Write down your steps so that someone else could follow your steps to find the same number of coins in a pouch.

1.



2.

3.

4.

5.

C. Describe how you can check your answer. That is, how do you know you found the correct number of gold coins in each pouch?

D. Describe how you maintained equality at each step of your solutions in Questions A and B.

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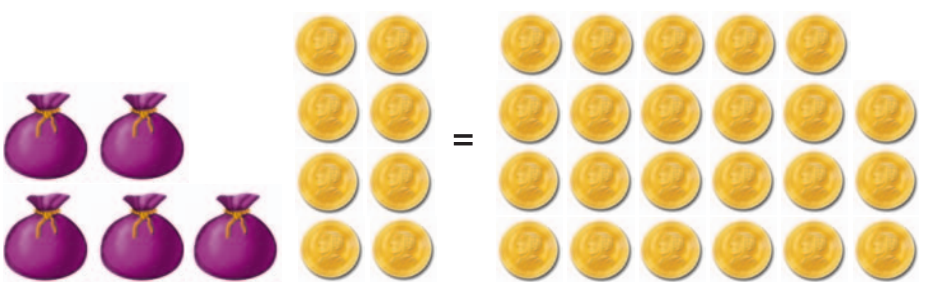
3.3 From Pouches to Variables

Throughout this unit, you have been solving problems that involve two variables. Sometimes the value of one variable is known, and you want to find the value of the other variable. The next problem continues the search for finding a value for a variable without using a table or graph. In this investigation, you are learning to use *symbolic* methods to solve a linear equation.

Getting Ready for Problem 3.3

The picture below represents another diplomatic pouch situation.



Because the number of gold coins in each pouch is unknown, we can let x represent the number of coins in one pouch and 1 represent the value of one gold coin.

- Write an equation to represent this situation.
- Use your methods from Problem 3.2 to find the number of gold coins in each pouch.
- Next to your work, write down a similar method using the equation that represents this situation.

Problem 3.3 Writing Equations

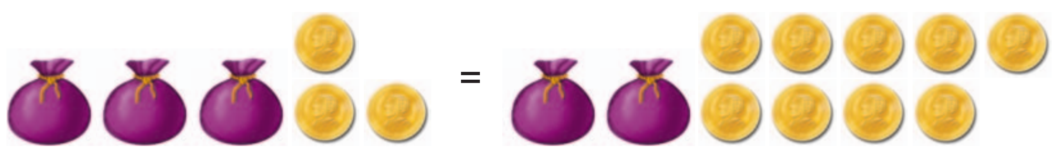
A. For each situation:

- Represent the situation with an equation. Use an *x* to represent the number of gold coins in each pouch and a number to represent the number of coins on each side.
- Use the equation to find the number of gold coins in each pouch.

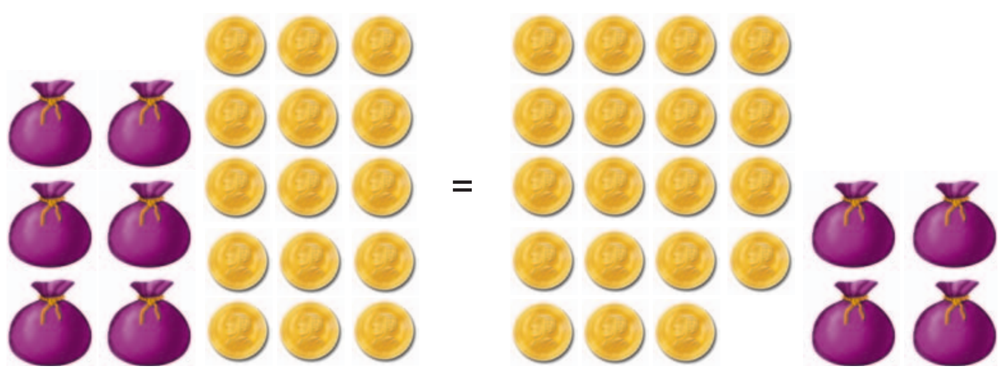
1.

2.

3.



4.



B. For each equation:

- Use your ideas from Question A to solve the equation.
- Check your answer.

1. $30 = 6 + 4x$

2. $7x = 5 + 5x$

3. $7x + 2 = 12 + 5x$

4. $2(x + 4) = 16$

C. Describe a general method for solving equations using what you know about equality.

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3.4 Solving Linear Equations

You know that to maintain an equality, you can add, subtract, multiply, or divide both sides of the equality by the same number. These are called the **properties of equality.** In the last problem, you applied properties of equality and numbers to find a solution to an equation.

So far in this investigation, all of the situations have involved positive numbers.

Does it make sense to think about negative numbers in a coin situation?

Getting Ready for Problem 3.4

- How do these two equations compare?

  $2x + 10 = 16$ $\qquad$ $2x - 10 = 16$

How would you solve each equation? That is, how would you find a value of x that makes each statement true?

- How do the equations below compare?

  $3x = 15$ $\qquad$ $-3x = 15$

  $3x = -15$ $\qquad$ $-3x = -15$

Find a value of x that makes each statement true.

Problem 3.4 Solving Linear Equations

Use what you have learned in this investigation to solve each equation.

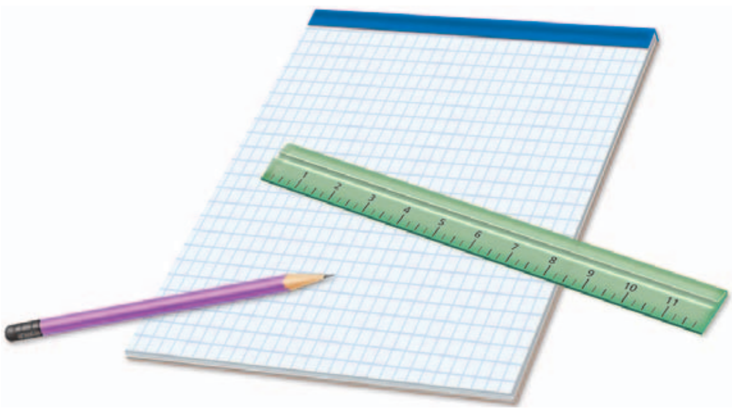
For Questions A–D, record each step you take to find your solution and check your answer.

A. **1.** $5x + 10 = 20$

2. $5x - 10 = 20$

3. $5x + 10 = -20$

4. $5x - 10 = -20$



B. **1.** $10 - 5x = 20$

2. $10 - 5x = -20$

C. **1.** $4x + 9 = 7x$

2. $4x + 9 = 7x + 3$

3. $4x - 9 = 7x$

4. $4x - 9 = -7x + 13$

D. **1.** $3(x + 2) = 21$

2. $-3(x - 5) = 2x$

3. $5(x + 2) = 6x + 3$

E. In all of the equations in Questions A–D, the value of x was an integer, but the solution to an equation can be any real number. Solve the equations below, and check your answers.

1. $5x + 10 = 19$

2. $5x + 10 = 9x$

3. $5x - 10 = -19$

4. $5x - 10 = -7x + 1$

F. **1.** Describe how you could use a graph or table to solve the equation $5x + 10 = -20$.

2. Suppose you use a different letter or symbol to represent the value of the unknown variable. For example, $5n + 10 = 6n$ instead of $5x + 10 = 6x$.

Does this make a difference in solving the equation? Explain.

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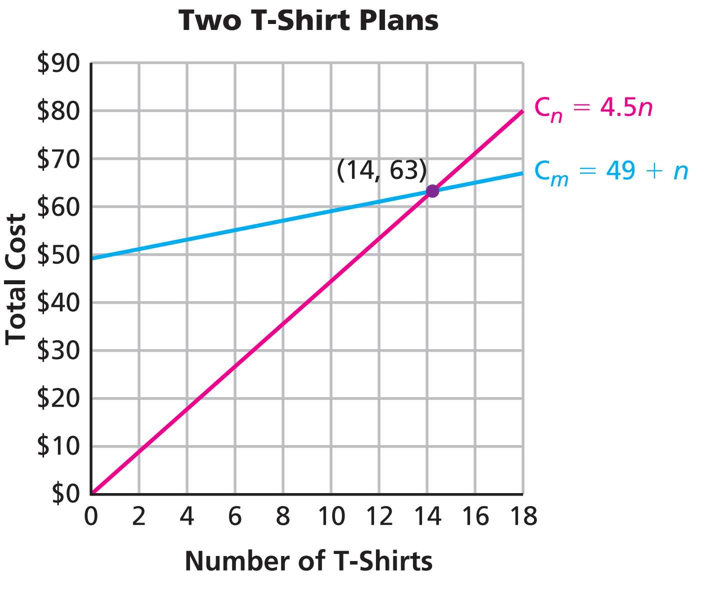
3.5 Finding the Point of Intersection

In Problem 2.3, you used the graphs (or tables) to find when the costs of two different plans for buying T-shirts were equal. The **point of intersection** of the two lines represented by the two graphs gives us information about when the costs of the two T-shirt plans are equal. The graphs of the two cost plans are shown below.



C_m is the cost for Mighty Tee.

C_n is the cost for No-Shrink Tee.

Getting Ready for Problem 3.5

- What information do the coordinates of the point of intersection of the two graphs give you about this situation?
- For what number(s) of T-shirts is plan C_m less than plan C_n? ($C_m < C_n$)
- Show how you could use the two equations to find the coordinates of the point of intersection of the two lines ($C_m = C_n$).

Problem 3.5 Finding the Point of Intersection

At Fabulous Fabian's Bakery, the expenses E to make n cakes per month is given by the equation $E = 825 + 3.25n$.

The income I for selling n cakes is given by the equation $I = 8.20n$.

A. In the equations for I and E, what information do the y-intercepts represent? What about the coefficients of n?

B. Fabian sells 100 cakes in January.

1. What are his expenses and his income?
2. Does he make a profit? Describe how you found your answer.

C. In April, Fabian's expenses are $5,700.

1. How many cakes does he sell?
2. What is the income for producing this number of cakes?
3. Does he make a profit? Explain.

D. The *break-even point* is when expenses equal income ($E = I$). Fabian thinks that this information is useful.

1. Describe how you can find Fabian's break-even point symbolically. Find the break-even point.
2. Describe another method for finding the break-even point.

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