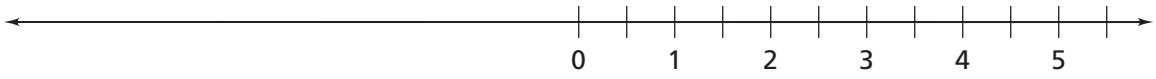
Investigation 1

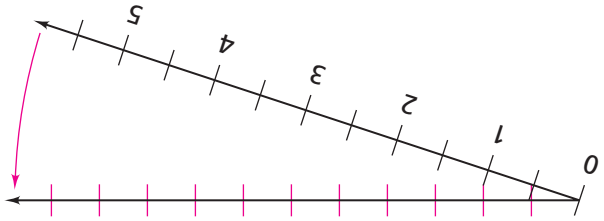
Extending the Number System

In your study of numbers, you have focused on operations (+, −, ×, and ÷) with whole numbers, fractions, and decimals. In this unit, you will learn about some important new numbers in the number system.

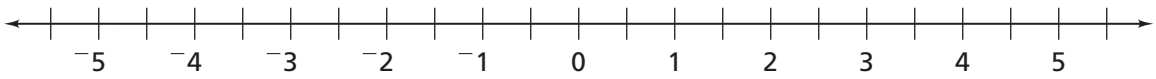
Suppose you start with a number line showing 0, 1, 2, 3, 4, and 5.



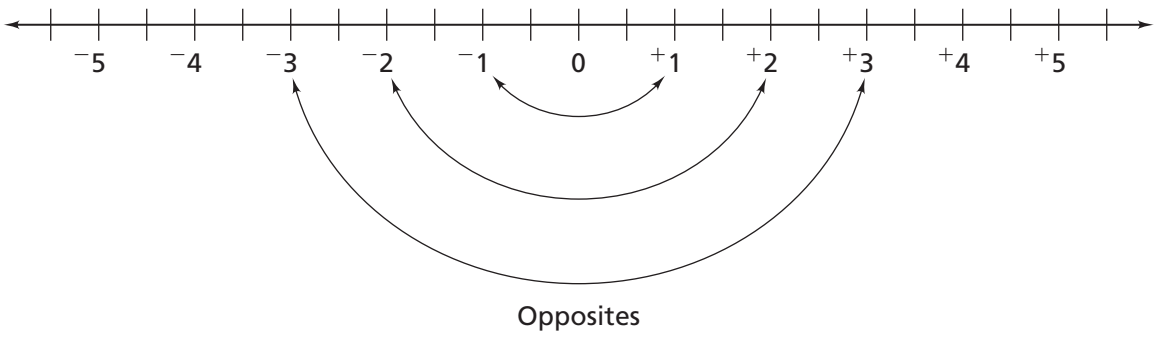
Take the number line and fold it around the zero point. Make marks on the left side of zero to match the marks on the right side.



Label the new marks with numbers that have negative signs (⁻). These numbers (to the left of 0) are **negative numbers.**



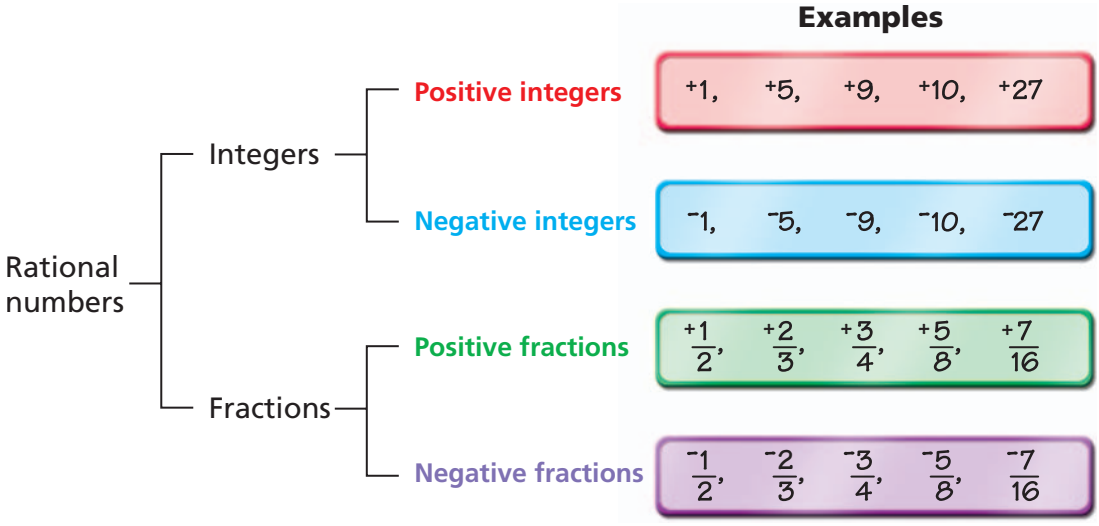
Each negative number is paired with a **positive number.** The numbers in the pair are the same distance from zero but in opposite directions on the number line. These number pairs are called **opposites.** You can label positive numbers with positive signs ($^+$).



Some subsets of the positive and negative numbers have special names. Whole numbers and their opposites are called **integers** ($^-4$, $^-3$, $^-2$, $^-1$, 0, $^+1$, $^+2$, $^+3$, $^+4$).

Fractions also have opposites. For example, $^+\frac{1}{2}$ and $^-\frac{1}{2}$ are opposites.

Positive and negative integers and fractions are called rational numbers. **Rational numbers** are numbers that can be expressed as one integer divided by another integer.



In mathematical notation, you can write a positive number with a raised plus sign ($^{+}150$) or without any sign (150). You can write a negative number with a raised minus sign ($^{-}150$). To avoid confusion with operation signs, it is common to use raised signs.

Many calculators have a special negative number key [(-)]. When you press 5 [−] [(-)] 2 , the calculator shows "5 − $^{-}$2."

Getting Ready for Problem 1.1

- Where would the following pairs of numbers be located on the number line?

  $^{+}7$ and $^{-}7$

  $^{+}2.7$ and $^{-}2.7$

  $^{-}3.8$ and $^{+}3.8$

  $-\frac{1}{2}$ and $+\frac{1}{2}$

  $4\frac{3}{4}$ and $-4\frac{3}{4}$

- If the same relationship holds true for all numbers, what would be the opposite of $^{-}1\frac{2}{3}$ and where would it be located?

1.1 Playing Math Fever

Ms. Bernoski's math classes often play Math Fever, a game similar to a popular television game show. The game board is shown. Below each category name are five cards. The front of each card shows a point value. The back of each card has a question related to the category. Cards with higher point values have more difficult questions.

Math Fever

Operations With Fractions	Similarity	Probability	Area and Perimeter	Tiling the Plane	Factors and Multiples
50	50	50	50	50	50
100	100	100	100	100	100
150	150	150	150	150	150
200	200	200	200	200	200
250	250	250	250	250	250

The game is played in teams. One team starts the game by choosing a card. The teacher asks the question on the back of the card. The first team to answer the question correctly gets the point value on the card. The card is then removed from the board. If a team answers the question incorrectly, the point value is subtracted from their score. The team that answers correctly chooses the next category and point value.

Problem 1.1 Using Positive and Negative Numbers

At one point in a game, the scores are as follows:

Super Brains	Rocket Scientists	Know-It-Alls
$^-300$	150	$^-500$

A. Which team has the highest score? Which team has the lowest score? Explain.

B. What is the difference in points for each pair of teams?

C. Use number sentences to describe two possible ways that each team reached its score.

D. The current scores are $^-300$ for Super Brains, 150 for Rocket Scientists, and $^-500$ for Know-It-Alls.

1. Write number sentences to represent each sequence of points. Start with the current score for each team.

a. Super Brains

Point Value	Answer
200	Correct
150	Incorrect
50	Correct
50	Correct

b. Rocket Scientists

Point Value	Answer
50	Incorrect
200	Incorrect
100	Correct
150	Incorrect

c. Know-It-Alls

Point Value	Answer
100	Incorrect
200	Correct
150	Incorrect
50	Incorrect

2. Now which team has the highest score? Which team has the lowest score?

3. What is the difference in points for each pair of teams?

E. The number sentences below describe what happens at a particular point during a game of Math Fever. Find each missing number. Explain what each sentence tells about a team's performance and overall score.

1. BrainyActs: $^-200 + 150 - 100 =$ ▢

2. MathSperts: $450 - 200 =$ ▢

3. ExCells: $200 - 250 =$ ▢

4. SuperMs: $^-350 +$ ▢ $= ^-150$

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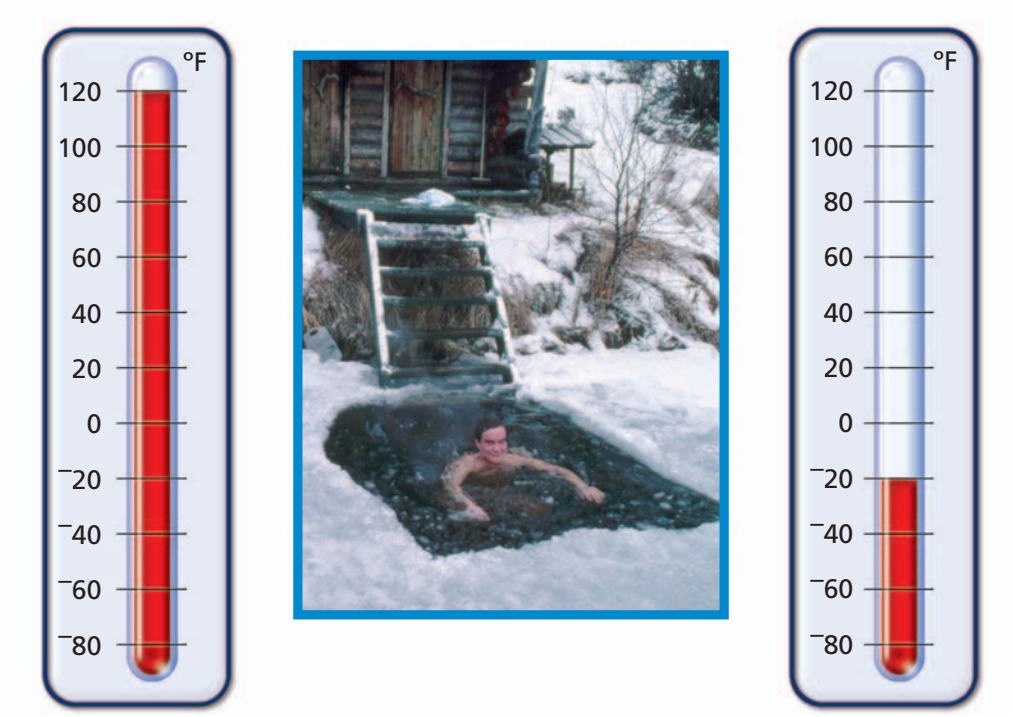
1.2 From Sauna to Snowbank

The record high and low temperatures in the United States are 134°F in Death Valley, California and ⁻80°F in Prospect Creek, Alaska. Imagine going from 134°F to ⁻80°F in an instant!

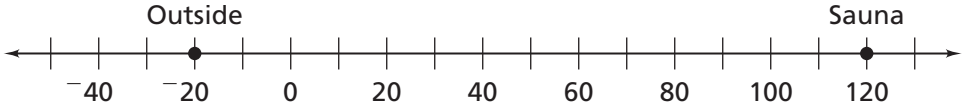
In Finland, people think that such temperature shocks are fun and good for your health. This activity is called sauna-bathing.

In the winter, Finnish people sit for a certain amount of time in sauna houses. The houses are heated as high as 120°F. Then the people run outside, where the temperature might be as low as ⁻20°F.

Inside the Sauna **Outside in Snow**



The two thermometers shown are similar to number lines. One horizontal number line can show the same information as the two thermometers.



On the number line, a move to the left is a move in a negative direction. The numbers decrease in value. A move to the right is a move in a positive direction. The numbers increase in value. On the thermometers, a move down means the number values decrease and the temperatures get colder. A move up means the number values increase and the temperatures get hotter.

Problem 1.2 Comparing and Ordering Positive and Negative Numbers

Sketch number lines to show your reasoning.

A. Order these temperatures from least to greatest.

$0°F$ $115°F$ $^{-}15°F$ $^{-}32.5°F$ $^{-}40°F$ $113.2°F$ $^{-}32.7°F$

B. For each pair of temperatures, identify which temperature is further from $^{-}2°F$.

1. $6°F$ or $^{-}6°F$?

2. $^{-}7°F$ or $3°F$?

3. $2°F$ or $^{-}5°F$?

4. $^{-}10°F$ or $7°F$?

C. Identify the temperature that is halfway between each pair of temperatures.

1. $0°F$ and $10°F$

2. $^{-}5°F$ and $15°F$

3. $5°F$ and $^{-}15°F$

4. $0°F$ and $^{-}20°F$

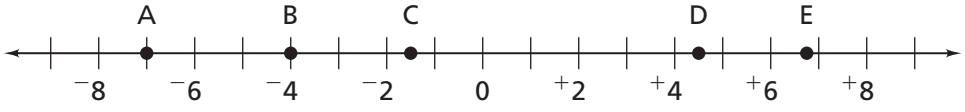
5. $^{-}8°F$ and $8°F$

6. $^{-}6°F$ and $6°F$

7. During one week, the high temperature was $60°F$. The halfway temperature was $15°F$. What was the low temperature?

D. Name six temperatures between $^{-}2°F$ and $^{+}1°F$. Order them from least to greatest.

E. 1. Estimate values for points A–E.



2. How does the number line help you find the smaller value of two numbers?

F. What are the opposites of these numbers?

1. 3 **2.** 7.5 **3.** $^{-}2\frac{2}{3}$

4. What is the sum of a number and its opposite?

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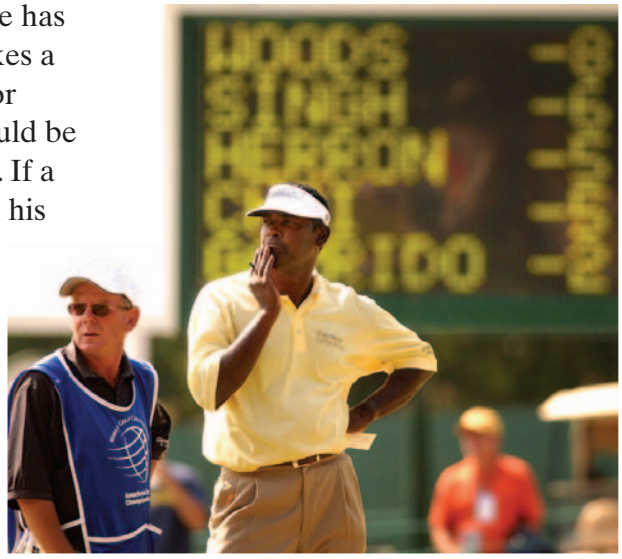
Did You Know?

In golf, scores can be negative. Each golf hole has a value called par. Par is the number of strokes a golfer usually needs to complete the hole. For example, a good golfer, like Vijay Singh, should be able to complete a par 4 hole in four strokes. If a golfer completes the hole in six strokes, then his or her score for that hole is "two over par" ($^{+}2$). If a golfer completes the hole in two strokes, his or her score is "two under par" ($^{-}2$). A player's score for a round of golf is the total of the number of strokes above or under par.



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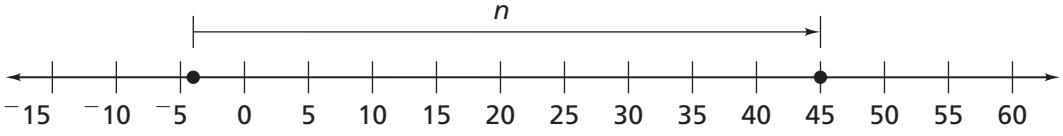
1.3 What's the Change?

The National Weather Service keeps records of temperature changes.

> The world record for fastest rise in outside air temperature occurred in Spearfish, South Dakota, on January 22, 1943. The temperature rose from $^{-}4$°F to 45°F in two minutes.

What was the change in temperature over that two minutes? How could you show this change, n, on the number line?

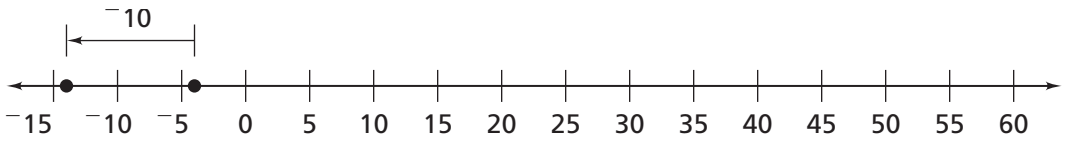


From $^{-}4$°F to 0°F is a change of $^{+}4$°F, and from 0°F to 45°F is a change of $^{+}45$°F. So the total change is $^{+}49$°F. The following number sentences show this.

$$^{-}4 + n = {}^{+}45$$
$$^{-}4 + {}^{+}49 = {}^{+}45$$

The sign of the change in temperature shows the direction of the change. In this case, $^{+}49$ means the temperature increased 49°F.

If the temperature had instead dropped 10° from $^{-}4$°F, you would write the change as $^{-}10$°F.



$$^{-}4 + {}^{-}10 = n$$
$$^{-}4 + {}^{-}10 = {}^{-}14$$

Problem 1.3 Using a Number Line Model

Sketch number lines and write number sentences for each question.

A. A person goes from a sauna at 120°F to an outside temperature of $^{-}20$°F. What is the change in temperature?

B. The temperature reading on a thermometer is 25°F. In the problems below, a positive number means the temperature is rising. A negative number means the temperature is falling. What is the new reading for each temperature change below?

1. $^{+}10$°F **2.** $^{-}2$°F **3.** $^{-}30$°F

C. The temperature reading on a thermometer is $^{-}15$°F. What is the new reading for each temperature change?

1. $^{+}3$°F **2.** $^{-}10$°F **3.** $^{+}40$°F

D. What is the change in temperature when the thermometer reading moves from the first temperature to the second temperature? Write an equation for each part.

1. 20°F to $^{-}10$°F **2.** $^{-}20$°F to $^{-}10$°F

3. $^{-}20$°F to 10°F **4.** $^{-}10$°F to $^{-}20$°F

5. 20°F to 10°F **6.** 10°F to 20°F

E. The temperature was $^{-}5$°F when Sally went to school on Monday. The temperature rose 20°F during the day, but fell 25°F during the night. A heat wave the next day increased the temperature 40°F. But an arctic wind overnight decreased the temperature 70°F! What was the temperature after the 70° decrease?

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1.4 In the Chips

When business records were kept by hand, accountants used red ink for expenses and black ink for income. If your income was greater than your expenses you were "in the black." If your expenses were greater than your income you were "in the red."

Julia has this problem to solve:

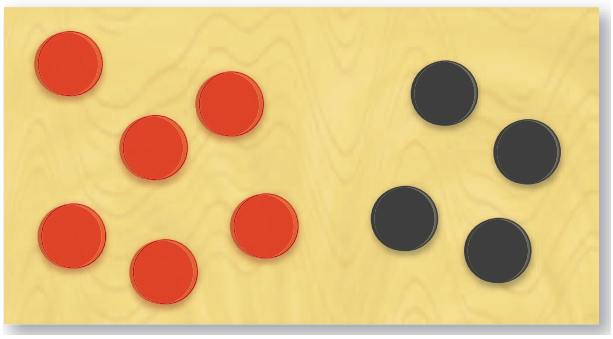
> Linda owes her sister \$6 for helping her cut the lawn. She earns \$4 delivering papers with her brother. Is she "in the red" or "in the black"?

Getting Ready for Problem 1.4

Julia uses red and black chips to model income and expenses. Each black chip represents $^{+}1$ dollar of income. Each red chip represents $^{-}1$ dollar of income (expenses).

Julia puts chips on the board to represent the situation. She decides Linda is "in the red" 2 dollars, or $^{-}2$ dollars.

Julia's Chip Board



- Why do you think she concludes that $^{-}6 + {}^{+}4 = {}^{-}2$?
- What is another way to show $^{-}2$ on the board?

Find the missing part for each chip problem. What would be a number sentence for each problem?

	Start With	Rule	End With
1.	3 red chips	Add 5 black chip	■
2.	2 red chips, 1 black chip	Subtract 3 red chips	■
3.	6 red chips	■	2 red chips
4.	■	Subtract 3 red chips	4 black chips

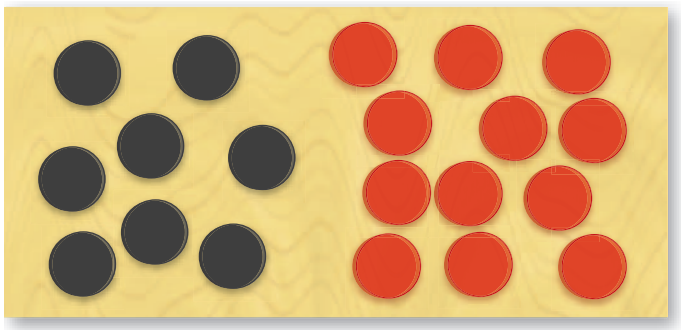
Problem 1.4 Using a Chip Model

Use ideas about black and red chips to answer each question. Then write a number sentence.

A. Give three combinations of red and black chips (using at least one of each color) that will equal each value.

1. 0 **2.** $^{+}12$ **3.** $^{-}7$ **4.** $^{-}125$

B. Use this chip board as the starting value for each part. Find the total value on each chip board.



1. original chip board

2. add 5 black chips

3. remove 5 red chips

4. remove 3 black chips

5. add 3 red chips

C. Cybil owes her sister $7. Her aunt pays her $5 to walk her dog. How much money does she have after she pays her sister?

D. Tate earns $10 mowing a lawn. He needs to pay $15 to rent his equipment. How much more money does he need to pay his rent?

E. Describe chip board displays that would match these number sentences. Find the results in each case.

1. $^{+}3 - {}^{+}2 = ■$ **2.** $^{-}4 - {}^{+}2 = ■$ **3.** $^{-}4 - {}^{-}2 = ■$

4. $^{+}7 + ■ = {}^{+}1$ **5.** $^{-}3 - {}^{+}5 = ■$ **6.** $■ - {}^{-}2 = {}^{+}6$

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